MODELING THE MORTGAGE LOAN PLACEMENT DECISIONS*

R. Stephen Sears, Assistant Professor, Department
 of Finance
Mike E. Miles, University of North Carolina at
 Chapel Hill
Arthur S. Estey, University of North Carolina at
 Chapel Hill

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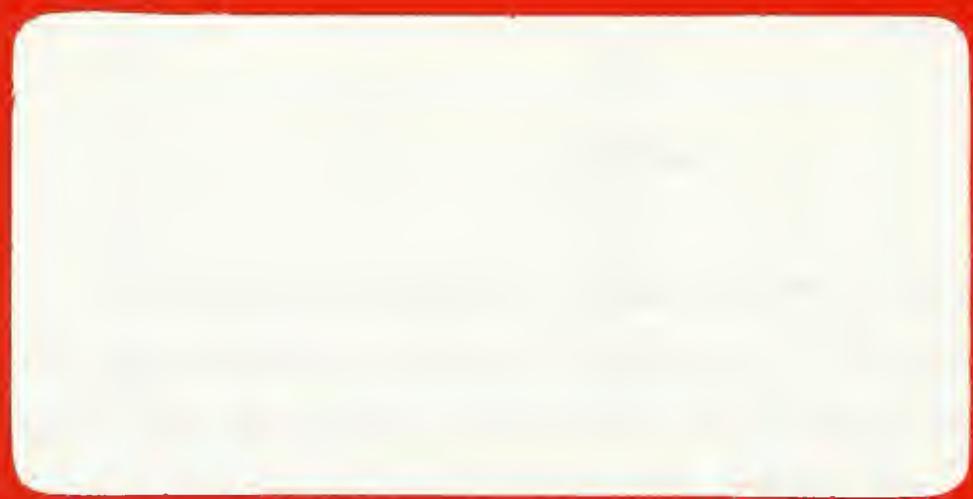MODELING THE MORTGAGE LOAN PLACEMENT DECISIONS*

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Summary

   This paper describes the mortgage banking function and the risk-return
perspective of the decision maker. The decision environment is expressed in
a straightforward, though quite extensive, dynamic programing model. While
solution of the complete model involving three-part state characterization
and multiple sources of uncertainty is beyond current technology, a reduced
version which replaces certain decision points with policy constraints is
presented and tested. The reduced model is shown to be a workable management
tool, while the complete model can serve as a basis for further research.

The Problem

The clearest presentation of the mortgage placement problem is through the mortgage banking firm (MBF) which is a non-depository financial intermediary whose principal activity is the origination and servicing of mortgage loans.[1] The firm operates by taking loan applications, committing funds to borrowers, closing the loans with finds normally borrowed through bank lines of credit and finally selling a package of loans to a permanent investor. Through this sequence of activities, the MFB seeks to maximize the discounted present value of a series of cash flows.

The cash flows derive from five distinct mortgage banking functions: (1) loan origination, (2) warehousing, (3) float, (4) servicing, and (5) marketing. Cash flows from loan origination are usually negative and equal the difference between the origination fee charged the borrower and the origination costs (mainly personnel time) incurred by the MBF. Loans held in the MBF's portfolio provide warehousing cash flows. The value of these cash flows depends on the interest rate differential between the long term rates received on the loans and the short term rate paid on the firm's commercial bank line of credit. Warehousing cash flows are usually positive when the yield curve slopes upward and are normally negative when the yield curve is downward-sloping. Float cash flows are always positive and represent the time value of money involved in holding interm payments by borrowers (principal, interest, insurance and taxes) prior to remittance to final lenders, insurance companies, and the government. The fee charged for "handling loan collections" for the final lender, less the cost of such operations, constitutes the servicing cash flow and is usually positive. The marketing cash flows are the gain or loss on the resale of mortgages to final lenders.

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[1] Unlike savings and loans, commercial banks and other mortgage lenders, the MBF has no deposits and no significant sources of revenue beyond mortgage banking. Consequently the MBF is the purest case of the entrepreneur in the field and offers a clear basis for the presentation of the mortgage loan placement decision.
When incorporated with other balance sheet considerations, the model developed in this paper is also appropriate for other mortgage lenders.

Historically, the MBF has sought to maximize loan production, thereby maximizing servicing revenue (which is the key profit item). The firm operates subject to two constraining factors: (1) limited credit lines and (2) fluctuating interest rates.

The first factor implies that loans must be continually "rolled over" to permit additional originations which will generate increased servicing revenue. Thus loans must be not only originated, but also sold. The MBF can sell either from inventory or, more frequently, by first securing purchase commitments from permanent lenders and then delivering loans to the permanent lenders against these commitments. Commitments have a limited life and can call for either mandatory or optional delivery by the MBF of some stated quantity of loans at a price that will provide the permanent investor with the yield specified in the commitment contract.

The size of the mortgage loan inventory which can be held at any point in time is a function of the MBF's equity and its line of credit (typically at a commercial bank). The amount that commercial banks will lend to a MBF is a function of the riskiness of the firm, primairly whether it has "good" purchase commitments for the vast majority of the loans in its portfolio. Since commitments are "good" or "bad" depending on market interest rates, fluctuating interest rates consequently necessitate continual commitment coverage adjustment. Adverse changes in interest rates render some of the MBF commitments virtually worthless by making delivery possible only at a substantial marketing loss. Hence, to maintain the necessary coverage, the firm must purchase additional commitments as old ones expire or are exercised, as the firm originates new loans, and as changes in interest rates reduce the value of its existing commitments.

Clearly, the decision environment of the MBF is more complex than simply "maximizing loan production." The firm must repeatedly make three interrelated, yet distinct, decisions in a world filled with uncertainty.

First, the firm must choose what volume of loans to originate. Second, the level of commitment coverage and the partitioning of the coverage among the placement alternatives must be determined.[2] Finally, delivery decisions must be made (i.e., the MBF must choose which commitments to exercise and which loans to deliver) thereby creating the marketing gain or loss.

The decisions depend on not only the MBF's current portfolio (loans as well as commitments), but also its expectations for future interest rates and mortgage demand. Moreover, the typical firm perfers covered originations, and, consequently, reduced exposure to interest rate fluctuations. Such a policy, however, requires the firm to purchase commitments without knowing the interest rate on its future originations. Furthermore, since some commitments are acquired through competitive sealed-bid auctions, the MBF cannot always be assured of obtaining all the commitments it seeks. Thus, even when originating loans against previously obtained commitments, there are two sources of uncertainty--the market determined rate on new originations and the outcome of the commitment auction. In addition, the firm must administer its origination and delivery decisions without violating its credit line agreement. Finally, the commitment decisions themselves must also be monitored to avoid excessive interest rate risk.[3]

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[2] The alternatives include the FNMA (standby convertibles and, more importantly, biweekly free market system auctions for both government insured and conventional loans in both competitive and noncompetitive modes), GNMA (negotiated commitments), FHLMC (weekly auction), privately guaranteed mortgage pass-throughs (major institutions only), and traditional private placements. For a fuller discussion and bibliography, see Sears, R. Stephen, "A Market Placement Model for the Mortgage Banker," Ph.D. dissertation, The University of North Carolina at Chapel Hill, 1979.

[3] Note that even optional commitments do not eliminate all risk. If interest rates fall, the firm can deliver and solve the volume problem but suffer a marketing loss. Alternatively, the firm can choose not to deliver and avoid the interest rate risk (the marketing loss), but in the process lose the solution to the volume problem.

Since the firm must make today's decisions in light of its expectations concerning the future, the entire process is dynamic. Decisions are not made in isolation, but as part of an on-going sequential decision process.

THE COMPLETE MODEL

The Sequence of Events

Every other week, the Federal National Mortgage Association (FNMA) holds a sealed-bid auction where mortgage bankers compete to purchase optional commitments. Other commitments can be purchased in negotiated transactions through the Government National Mortgage Association (GNMA) or with permanent lenders.[4]

To formulate a dynamic model, let each time period represent two weeks --the frequency of the FNMA auctions. Within each period, let events occur in four stages.

First, the firm places its bid(s) in the FNMA auction. The firm must make its bidding decision without knowing (though with certain expectations concerning) (a) the results of the FNMA auction, (b) the rates available this period on all other commitments, (c) the rates at which new loans can be originated this period, and (d) the rate to be charged by the commercial bank for this period's financing.

Second, the uncertainties are resolved. The firm learns the results of the FNMA auction--both this period's interest rates on FMNA commitments and the acceptance or rejection of the firm's bid(s). The interest rates on other placement alternatives for this period similarly become known.

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[4] These are the major placement alternatives. For additional alternatives and descriptions, see Sears, op. cit.

Finally, new loans are originated, the firm's decisions result in a
profit or loss for the period, the company's balance sheet is adjusted appro-
priately, and the system moves to the beginning of a new two-week period
when the MBF must again decide how to bid in the up-coming FNMA auction.[5]

Model Specification

As formulated below, the model requires decision variables, descriptive
variables, and several parameters. The decision variables relate to bidding
in the FNMA auctions, purchasing other commitments, loan delivery against
existing commitments, and closing of new loans. The three latter groups
of decision variables depend on q. Other variables, most of which depend
on q, describe new loans, interest rates, and balance sheet conditions.
A set of parameters, specifying cost factors and operating constraints,
complete the list of model elements. After an explanation of the superscript
and subscript notation used, the following sections define these quantities.

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[5] In the discussion that follows, frequent use is made of the symbol q.
This variable represents the state of the world after the several uncertainties
are resolved. Specifically, q_t is the set of information that becomes
available to the MBF in period t after it has bid in the FNMA auction.
Let Q denote the set of all possible realizations q.
    The MBF uses the information set q in deciding what additional com-
mitments to purchase after learning the results of the FNMA auction and in
making its delivery decisions. In a given period, the realized value of
q determines what commitments, if any, the firm succeeded in buying
through the FNMA auction. From the standpoint of the MBF q also deter-
mines what all the interest rates are for the period and how much demand
for new loans exists in the period.
    For all q in Q, let $P(q)$ denote the probability that q occurs.
Of course, $P(q) \geq 0$, for all q, and $\sum_{q \text{ in } Q} P(q) = 1$.
    The model assumes that the MBF knows the distirbution $P(q)$ before
making the bidding decision at the start of each period. That is, the
MBF is assumed to have a good idea about what interest rates will be over
the next two weeks and about what the low bid will be in the upcoming
FNMA action.

With the exception of $A^i_{\ell,t}(q_t)$, $OC^i_{\ell t}(q_t)$, and $DP^i_{j,k,\ell,m,p,c,t}(RC^i_{j,k,m,p}(q_p), RV^i_{\cdot,c}(q_c))$ for all i, j, k, ℓ, m, p, c, and t, all of the variables and parameters are constrained to be nonnegative.

Superscript and Subscripts

$i = 1,2$	Superscript identifying loan classification: $i=1$ refers to FHA/VA loans; $i=2$ refers to conventional loans.[6]
$j = 1,2,3$	Subscript identifying commitment classification: $j=1$ refers to FNMA commitments; $j=2$ refers to mandatory commitments; $j=3$ refers to non-FNMA optional commitments.
$k = 1,\ldots,K_j$	For $j = 1, 2, 3$, subscript identifying different commitment devices within a given classification. For FNMA commitments, $K_1=5$, since the firm may enter up to 5 bids in each period's FNMA ACUTION.
$\ell = 1,\ldots,L$	Subscript identifying loan category (property characteristics, etc.).
$m = 1,\ldots,M$	Subscript identifying period in which commitments expire.
$p = 0,\ldots,t$	Subscript identifying period in which a commitment was purchased.
$c = 0,\ldots,t$	Subscript identifying period in which a loan was closed.
t	Subscript identifying the current period.

Decision Variables

$W^i_{k,t}$, $i=1,2$, $k=1,\ldots,5$	= face amounts bid in FNMA auctions. (The firm can enter up to 5 bids of up to $3 MM each. There are separate auctions for government insured and conventional loans.)
$RW^i_{k,t}$, $i=1,2$, $k=1,\ldots,5$	= interest rates bid in FNMA auctions.
$C^i_{j,k,m,t}(q_t)$, $i=1,2$, $j=2,3$, $k=1,\ldots,K_j$, $m=t+1,\ldots,M$	= face amounts of non-FNMA commitments purchased. (See Footnote 5 for a discussion of q.)
$X^i_{j,k,\ell,m,p,c,t}(q_t)$, $i=1,2$, $j=1,2,3$, $k=1,\ldots,K_j$, $\ell=1,\ldots,L$, $m=t,\ldots,M$, $p,c=0,\ldots,t-1$	= face amounts of commitments exercised via loan delivery.

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[6] Loans are assumed to be homogeneous with respect to all characteristics except category, whether FHA/VA or conventional, and interest rate.

$V^i_{\ell,t}(q_t)$, $i=1,2$, $\ell=1,\ldots,L$ = face amounts of loans closed.

Descriptive Variables

$C^i_{1,k,m,t}(q_t, RW^i_{k,t})$, $i=1,2$, $k=1,\ldots,5$, $m=t+8$ = face amounts of FNMA commitments bought through auction. (All FNMA commitments have initial 4-month, i.e., 8-period, maturities.)

$RC^i_{j,k,m,t}(q_t)$, $i=1,2$, $j=2,3$, $k=1,\ldots,K_j$, $m=t,\ldots,M$ = interest rates on non-FNMA commitments. (The interest rate on FNMA commitments purchased through accepted bid k is $RW^i_{k,t}$.)

$RV^i_{\ell,t}(q_t)$, $i=1,2$, $\ell=1,\ldots,L$ = interest rate on new loans closed.

$D^i_{\ell,t}(q_t)$, $i=1,2$, $\ell=1,\ldots,L$ = face amount of new loan demand.

$A^i_{\ell,t}(q_t)$, $i=1,2$, $\ell=1,\ldots,L$ = after-tax present value coefficient for servicing revenue to be earned per dollar face amount of loans closed.

$RB_t(q_t)$ = interest rate charged by commercial bank on firm's borrowing.

$B_t(q_t)$ = face amount of firm's outstanding borrowings.

$EQ_t(q_t)$ = firm's equity position.

$DP^i_{j,k,\ell,m,p,c,t}(RC^i_{j,k,m,p}(q_p), RV^i_{\ell,c}(q_c))$, $i=1,2$, $j=1,2,3$, $k=1,\ldots,K_j$, $\ell=1,\ldots,L$, $m=t,\ldots,M$, $p=0,\ldots,t-1$, $c=0,\ldots,t-1$ = discount points paid per dollar face amount of loans delivered.

$FL^i_{\ell,t}(q_t)$, $i=1,2$, $\ell=1,\ldots,L$ = after-tax present value coefficient for float revenue to be earned per dollar face amount of loans closed.

$F^i_{\ell,t}(q_t)$, $i=1,2$, $\ell=1,\ldots,L$ = average annual float income per dollar face amount of loans.

Parameters

$OC^i_{\ell,t}(q_t)$, $i=1,2$, $\ell=1,\ldots,L$ 　　= origination cost per dollar face amount of loans closed.[7]

$BC^i_{k,t}$, $i=1,2$, $k=1,\ldots,5$ 　　= bidding cost per dollar face amount of FNMA commitments sought.

$CC^i_{j,k,m,t}$, $i=1,2$, $j=1,2,3$, $k=1,\ldots,K_j$, $m=t+1,\ldots,M$ 　　= commitment cost per dollar face amount of commitments purchased.

$DC^i_{j,k,\ell,m,p,t}$, $i=1,2$, $j=1,2,3$, $k=1,\ldots,K_j$, $\ell=1,\ldots,L$, $m=t+1,\ldots,M$, $p=0,\ldots,t-1$ 　　= delivery cost per dollar face amount of commitments exercised.

CL^i 　　= the firm's coverage exposure limit for loans.

BL 　　= the firm's maximum borrowing limit.

EL 　　= the firm's minimum equity limit.

CA 　　= the firm's (fixed) amount of cash.

TX 　　= the firm's marginal tax rate.

a , $0 < a < 1$ 　　= one-period discount factor applicable to after-tax cash flows.[8]

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[7] Origination costs depend on q because there may exist discount points on closing when market interest rates exceed statutory ceilings.

[8] The discount factor, a, is not a function of q because each period is assumed to be only two weeks long. State-dependent discount factors should be considered for models with significantly longer periods.

The State of the Firm

In any period t, the pre-auction position of the firm is jointly defined by its loans (inventory), commitments from permanent lenders, cash, equity, and bank borrowings. The loan position includes both the amounts and yields of all loans closed in periods $0, 1, \ldots, t-1$, remaining on the balance sheet at the beginning period t. The amounts and rates of all commitments owned at the beginning of the period constitute the commitment position. Cash, equity, and bank borrowings are represented by their balance sheet amounts. Also important are the past values of q.

The following quantities, then, together describe the state of the firm at the beginning of period t.

$$(1) \quad U^i_{\ell,n,t-1} = V^i_{\ell,n}(q_n) - \sum_{z=n+1}^{t-1} \sum_{j=1}^{3} \sum_{k=1}^{K_j} \sum_{m=1}^{M} \sum_{p=0}^{t-1} X^i_{j,k,\ell,m,p,n,z}(q_z)$$

for $i=1,2$, $\ell=1,\ldots,L$, and $n=0,\ldots,t-1$

$$(2) \quad RV^i_{\ell,n}(q_n) \quad \text{for } i=1,2, \ \ell=1,\ldots,L, \text{ and } n=0,\ldots,t-1$$

$$(3) \quad H^i_{1,k,m,n,t-1} = C^i_{1,k,m,n}(q_n, RW^i_{k,n}) - \sum_{\ell=1}^{L} \sum_{c=0}^{t-1} \sum_{z=n+1}^{t-1} X^i_{1,k,\ell,m,n,c,z}(q_z)$$

for $i=1,2$, $k=1,\ldots,5$, $m=t,\ldots,M$, $n=0,\ldots,t-1$

$$(4) \quad H^i_{j,k,m,n,t-1} = C^i_{j,k,m,n}(q_n) - \sum_{\ell=1}^{L} \sum_{c=0}^{t-1} \sum_{z=n+1}^{t-1} X^i_{j,k,\ell,m,n,c,z}(q_z)$$

for $i=1,2$, $j=2,3$, $k=1,\ldots,K_j$, $\ell=1,\ldots,L$, $m=t,\ldots,M$, $n=0,\ldots,t-1$

$$(5) \quad RC^i_{j,k,m,n}(q_n) \quad \text{for } i=1,2, \ j=1,2,3, \ k=1,\ldots,K_j, \ m=t,\ldots,M, \ n=0,\ldots,t-1$$

(6) q_n for $n=0,\ldots,t-1$

(7) CA

(8) $B_{t-1}(q_{t-1})$

(9) $EQ_{t-1}(q_{t-1})$

Expressions (1) and (2) describe the firm's loan portfolio at the
beginning of period t (the end of period t-1). The loans of category
i , type ℓ , closed in period n , remaining in the firm's portfolio at
the beginning of period t , are all those closed in period n , less the
ones that have been delivered to satisfy exercised commitments. Expression
(2) records the interest rates on each type of loan.

Expressions (3), (4), and (5) refer to the company's outstanding
commitments. The commitments presently owned by the firm are all those
it has purchased, minus those that have expired, minus those that have
been exercised. The interest rates on the remaining commitments are recorded
in expression (5).

The cash, debt, and equity positions of the firm at the beginning
of period t are given in expressions (7), (8), and (9).

The Objective Function

The assumed goal of the MBF is to maximize the expected dis-
counted present value of the incremental cash flows resulting from
loan closings, commitment purchases, and delivery decisions.

Recall that the MBF makes two sets of decisions. First it bids
in the FNMA auction (amounts and rates $W_{k,t}^i$ and $RW_{k,t}^i$, respectively,
where k=1,...,5 and i=1,2). After the auction it makes other commitment,

delivery, and loan closing decisions. Let x_t and y_t denote the vectors of decision variables chosen in period t before and after the FNMA auction, respectively.

Let s_t denote the state of the system at the beginning of period t as described above. Given s_t, note that x_t, y_t, and q_t jointly determine s_{t+1}, the state of the firm at the beginning of period $t+1$. Denote this relationship by $s_{t+1} = g(s_t, x_t, y_t, q_t)$.

To illustrate the form of the objective function, define the following quantities.

$h(s_t, x_t)$ = the expected contribution to the objective function of the bidding activities in period t, given pre-auction state s_t and bid decisions x_t.

$Z(s_t, x_t, y_t, q_t)$ = the contribution to the objective function of the post-auction decisions in period t given pre-auction state s_t, bidding decisions x_t, information set realization q_t, and post-auction decisions y_t.

$f_t(s_t)$ = the value of the objective function for the optimal policy from period t onward, given that the firm starts period t in state s_t.

The objective of the MBF is to select x_t and y_t to solve

$$(10) \quad f_t(s_t) = \max_{x_t, y_t} \left\{ h(s_t, x_t) + \sum_{q_t \text{ in } Q} P(q_t) \left\{ Z(s_t, x_t, y_t, q_t) + a f_{t+1}[g(s_t, x_t, y_t, q_t)] \right\} \right\}.$$

Equation (10) contains the cash flows resulting from the firm's decisions in period t : $CF_t(s_t, x_t, y_t) = h(s_t, x_t) + \sum_{q_t} P(q_t) Z(s_t, x_t, y_t, q_t)$.

In terms of the variables defined earlier, these cash flows are:

$$(11) \quad CF(s_t, x_t, y_t) = -(1-TX) \sum_{i=1}^{2} \sum_{k=1}^{5} BC_{k,t}^{i} W_{k,t}^{i}$$

$$+ \sum_{q_t \text{ in } Q} P(q_t) \{ -(1-TX) \sum_{i=1}^{2} [\sum_{k=1}^{5} \sum_{m=t+1}^{M} CC_{1,k,m,t}^{i} C_{1,k,m,t}^{i}(q_t, RW_{k,t}^{i})$$

$$+ \sum_{j=2}^{3} \sum_{k=1}^{K_j} \sum_{m=t+1}^{M} CC_{j,k,m,t}^{i} C_{j,k,m,t}^{i}(q_t)]$$

$$+ \sum_{i=1}^{2} \sum_{\ell=1}^{L} [A_{\ell,t}^{i}(q_t) + FL_{\ell,t}^{i}(q_t) - (1-TX)OC_{\ell,t}^{i}(q_t) - 1] V_{\ell,t}^{i}(q_t)$$

$$+ \frac{1}{26}(1-TX) \sum_{i=1}^{2} \sum_{\ell=1}^{L} [RV_{\ell,t}^{i}(q_t) V_{\ell,t}^{i}(q_t) + \sum_{n=1}^{t-1} RV_{\ell,n}^{i}(q_n) U_{\ell,n,t-1}^{i}]$$

$$+ [1 - \frac{1}{26}(1-TX)RB_t(q_t)][B_t(q_t) - B_{t-1}(q_{t-1})]$$

$$+ \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{K_j} \sum_{\ell=1}^{L} \sum_{m=t}^{M} \sum_{p=0}^{t-1} \sum_{c=0}^{t-1} [1 - (1-TX)DC_{j,k,\ell,m,p,t}^{i}$$

$$- (1-TX)DP_{j,k,\ell,m,p,c,t}^{i}(RC_{j,k,m,p}^{i}(q_p), RV_{\ell,c}^{i}(q_c))]X_{j,k,\ell,m,p,c,t}^{i}(q_t)\}$$

The first term in equation (11) represents the bidding costs from participation in the FNMA auctions in period t . The first two terms inside the braces are the costs of the commitments acquired in period t . The next expression combines servicing revenue, float (including escrow balances),[9] origination costs, and the outlay of principal on new loans made in period t . The following three terms encompass the warehousing

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[9] Ideally, the formulation should also capture the differential impacts that loan delivery has upon estimated float cash flows. Servicing agreements vary among secondary market purchasers as to when mortgage payments are forwarded. Estimated float cash flows should ideally be adjusted according to which placement alternative is utilized.

cash flows and express, respectively, interest income on new loans, interest income on loans already in the portfolio, and funds generated or lost through the change in the firm's debt level, net of the associated interest cost.[10] The balance of the equation represents the marketing cash flows--those generated by exercising commitments and selling loans--net of delivery charges and discount points paid, if any.

By considering only those elements of (11) that contribute to the profit or loss shown on the MBF income statement, a similar expression exists for the firm's income, $Y_t(s_t, x_t, y_t, q_t)$. (In the equation below, the figure .00375, representing an average annual servicing income per dollar face amount of loans, was obtained through interviews with mortgage bankers.)

$$(12) \quad Y_t(s_t, x_t, y_t, q_t) = (1-TX) \{ - \sum_{i=1}^{2} \sum_{k=1}^{5} BC_{k,t}^i W_{k,t}^i$$

$$- \sum_{i=1}^{2} [\sum_{k=1}^{5} \sum_{m=t+1}^{M} CC_{1,k,m,t}^i C_{1,k,m,t}^i (q_t, RW_{k,t}^i)$$

$$+ \sum_{j=2}^{3} \sum_{k=1}^{K_j} \sum_{m=t+1}^{M} CC_{j,k,m,t}^i C_{j,k,m,t}^i (q_t)]$$

$$+ \sum_{i=1}^{2} \sum_{\ell=1}^{L} \{ \frac{1}{26} (.00375 + F_{\ell,t}^i(q_t)) - OC_{\ell,t}^i(q_t)] V_{\ell,t}^i(q_t)$$

$$+ \frac{1}{26} \sum_{i=1}^{2} \sum_{\ell=1}^{L} \sum_{n=0}^{t-1} U_{\ell,n,t-1}^i RV_{\ell,n}^i(q_n) + \frac{1}{26} \sum_{i=1}^{2} \sum_{\ell=1}^{L} V_{\ell,t}^i(q_t) RV_{\ell,t}^i(q_t)$$

$$- \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{K_j} \sum_{\ell=1}^{L} \sum_{m=t}^{M} \sum_{p=0}^{t-1} \sum_{c=0}^{t-1} [DP_{j,k,\ell,m,p,c,t}^i (RC_{j,k,m,p}^i(q_p), RV_{\ell,c}^i(q_c))$$

$$+ DC_{j,k,\ell,m,p,t}^i] - \frac{1}{26} RB_t(q_t)[B_t(q_t) - B_{t-1}(q_{t-1})] \}$$

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[10] This formulation presumes the warehousing period to be short enough to justify the use of face amounts in the computation of warehousing income. In other words, the amount of principal reduction during the warehousing period is negligible. This seems realistic in light of current mortgage banking practice where a typical warehousing period is 60 days. The fraction (1/26) translates annual rates into those appropriate to a two-week period.

Equation (12) is useful in projecting the capital structure of the firm at the end of period t. The expected profit from a particular strategy (x_t, y_t) can be written $E_{x_t, y_t}[Y_t]$, where the expectation is taken with respect to the possible values of q_t. Given $EQ_{t-1}(q_{t-1})$, the equity position of the firm at the end of period $t-1$, the expected equity position one period later is $E[EQ_t] = EQ_{t-1}(q_{t-1}) + E_{x_t, y_t}[Y_t]$, if (x_t, y_t) are chosen in period t and dividends are ignored.

The Constraint Set

When performing the maximization (10), the MBF finds itself subject to several constraints. In addition to the nonnegativity restrictions noted earlier, the firm must satisfy a number of legal and economic requirements. In particular, the MBF maximizes (10) subject to the following conditions.

(13) $H^i_{2,k,t,p,t} = \sum_{c=0}^{t-1} \sum_{\ell=1}^{L} X^i_{2,k,\ell,t,p,c,t}(q_t)$, for $i=1,2$, $k=1,\ldots,K_2$, $p=0,\ldots,t-1$

(14) $\sum_{\ell=1}^{L} \sum_{n=0}^{t} U^i_{\ell,n,t} - \sum_{j=i}^{3} \sum_{k=1}^{K_j} \sum_{m=t+1}^{M} \sum_{p=0}^{t} H^i_{j,k,m,p,t} \leq CL^i$, for $i=1,2$

(15) $V^i_{\ell,t}(q_t) \leq D^i_{\ell,t}(q_t)$ for $i=1,2$, $\ell=1,\ldots,L$

(16) $EQ_{t-1}(q_{t-1}) + E_{x_t, y_t}[Y_t] \geq EL$

(17) $CA + \sum_{i=1}^{2} \sum_{\ell=1}^{L} \sum_{n=0}^{t} U^i_{\ell,n,t} - EQ_{t-1}(q_{t-1}) - E_{x_t, y_t}[Y_t] \leq BL$

Equations (13) force the firm to deliver loans against expiring mandatory purchase commitments. The inequalities (14) ensure that at the end of each period, all the loans in the firm's portfolio are covered

by valid commitments, at least to within some acceptable tolerance CL . Line (15) notes that loan originations cannot exceed demand. The equity constraint (16) holds that the firm's expected income in period t must be sufficient to preserve stockholders' equity in an amount at least as large as the minimum acceptable, EL . Finally, (17) limits the amount of debt the firm can incur. The firm's balance sheet assets consist of cash and loans. The difference between total assets and the expected equity is the expected borrowing requirement, which must not exceed BL .

Analysis and Solution

Unfortunately, no solution to the model presented in (10) through (17) is readily at hand. The solution technique normally applied to sequential decision problems, such as (10), is dynamic programming. However, solution of a model such as the one offered here--with its multiple decisions within each period; large numbers of decision variables, dimensions, and random variables; intricate structure; and complex constraint set--is far beyond the present state of the art. What the model formulated above does provide is a general statement of the mortgage palcement problem with an explanation of the relationships among the different aspects of the problem in analytical terms. It offers a conceptual framework from which smaller, more tractable models can be constructed to focus on particular subproblems. The balance of this paper describes and analyzes one such model.[11]

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[11]Another promising alternative is to revise the state and transition definitions so as to conform to the conditions of Sobel's theorem which provides conditions under which the myopic solution to a dynamic problem is optimal. Unfortunately, large amounts of computer time are required to solve the resulting multivariable nonlinear problem in multidimensional space.

<center>### THE REDUCED MODEL</center>

The first decision the MBF faces each period is how to bid in
the upcoming FNMA auction. This decision is the focus of the model
analyzed below. The other decisions--purchase of other commitments, loan
closings, and loan delivery--are assumed to be made in accordance with
some exogenously specified policy rule. (Of course, the effects of different
policy rules on bidding decisions and cash flows can be tested by rerunning
the model.)

The model presented below is essentially a reduced version of (10)
through (17). The reduction occurs by making a number of simplifying
assumptions that render the model more tractable without eliminating
essential characteristics of the problem.

The model only considers FHA/VA loans. A typical MBF conducts the
majority of its business in this area. Moreover, until the Federal Home
Loan Mortgage Corporation auctions become fully open to mortgage bankers,
and until the privately-guaranteed pass-through gains a wider acceptance
among investors, FHA/VA loans will continue to command the lion's share
of the MBF's attention.

All FHA/VA loans are assumed to be homogeneous with respect to all
characteristics except interest rate and dollar face amount. Given the
borrower/lender/property requirements of the FHA and the VA, this is not
a very severe assumption.

The MBF may purchase two kinds of commitments, four-month optional
commitments through the FNMA auction or four-month optional commitments
through private negotiation. The typical MBF places most of its FHA/VA
loans through the FNMA.[13] All other commitment devices can reasonably

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[13] For conventional loans, high quality paper can often be marketed
privately at a lower yield than through the FNMA. (This is consistent with
the FNMA charter which provides that the FNMA's role should be supplemental
and not dominant in the market.) Consequently, mortgage banking subsidiaries

be condensed into one because (a) the net servicing revenue--the largest
profit item--is similar across all other FHA/VA loan placements, (b) com-
mitment costs are in the same range, and (c) yield differentials typically
are small.

Warehousing and float considerations are ignored. Warehousing issues
interact most directly with the delivery decision and float relates to dif-
ferent remittance policies among the different commitment alternatives.
Hence, the firm may reasonably be assumed to take these factors into account
in establishing the policy rule for the other decisions.

Three additional assumptions make the model far easier to manipulate
and analyze. First, the cash and equity positions of the firm are both
assumed to be constant. These components of the typical MBF's balance
sheet are relatively fixed - a large protion of profits are paid
out and bank lines remain fairly stable over the relevant time
horizon. Second, tax considerations are ignored. Their inclusion is a
straightforward extension of the model, but would complicate matters unneces-
sarily at this juncture. Third, the discount factor is assumed to be a
constant. Since one period in the model represents two weeks, the firm's
discount rate should not be expected to change much from period to period.

In each period, the policy rule must specify (a) how many loans to
close, (b) which commitments to exercise, (c) which loans to deliver against
exercised commitments, and (d) how much additional new commitment coverage
to purchase. For the purposes of this paper, new originations are assumed
to equal demand (an exogenous random variable). Commitments are first exercised
in an amount sufficient to maintain the loan portfolio at or below the firm's
credit limit, with expiring commitments being exercised first. Because
origination volume and loan turnover are so important, further deliveries
are assumed to be made to the extent that the firm can profitably make them.
Loans are delivered from the portfolio on a weighted-average interest rate
basis. Finally, the firm's "good" commitment coverage is computed and addi-
tional commitments are purchased as needed to maintain the uncovered portion

of the portfolio at or below a pre-determined maximum. (A "good" commitment is one whose interest rate is less than the current period's loan origination rate plus some exogenously input tolerance.) This "good coverage" rule penalizes the MBF if it bids too high in the FNMA auction. By bidding high, the firm can achieve a high probability that its bid will be accepted, but it purchases potentially "bad" coverage as a result.

In the initial period, then, the MBF chooses the amount and interest rate to bid in the upcoming FNMA auction. After the results of the auction become known, the policy rule is invoked to make delivery and commitment purchase decisions. Profit or loss is calculated, the state of the firm is updated, and the system advances to the next period. For decision-making purposes in a given period, it is assumed that the amount bid in the FNMA auctions in future periods is \$15,000,000 (the greatest total that the FNMA will allow) or the difference between the firm's current loan position plus expected new demand and its "good" coverage, whichever is less. Historically, negotiated commitments tend to be more expensive that those purchased through the FNMA auctions, so mortgage bankers strive to obtain as much of their coverage needs through the auctions as possible.[14]

The state of the firm at the beginning of period t, s_t, is defined by the union of five sets: (a) the loans on hand, $\{U_{\ell,n,t-1}: \ell=1,\ldots,L, n=0,\ldots,t-1\}$; (b) the interest rates on these loans, $\{RV_{\ell,n}(q_n): \ell=1,\ldots,L, n=0,\ldots,t-1\}$; (c) the unexpired, unexercised commitments, $\{H_{j,n,t-1}: j=1,2, n=0,\ldots,t-1\}$; (d) the interest rates on these commitments, $\{(RC_{1,n}, RC_{2,n}(q_n)): n=0,\ldots,t-1\}$; and (e) the information history, $\{q_n: n=0,\ldots,t-1\}$. Thus, the meanings of the components of s_t in this auction model are analogous to those of their counterparts in the larger problem.

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[14]This has been true historically, but at the time of this draft (October 1979), required GNMA yields are significantly more attractive.

Actions W_t and $RC_{1,t}$ (respectively, the amount and rate bid), together with the random outcome q_t, transform s_t into s_{t+1} as follows.

$$U_{\ell,n,t} = U_{\ell,n,t-1} - \frac{U_{\ell,n,t-1}}{t-1 \atop \sum\limits_{i=0} U_{\ell,i,t}} \sum_{j=1}^{2} \sum_{p=t-8}^{t-1} \sum_{c=0}^{t-1} X_{j,\ell,p,c,t}(s_t,W_t,RC_{1,t},q_t)$$

for $\ell=1,\ldots,L$ and $n=0,\ldots,t-1$

and $U_{\ell,t,t} = V_{\ell,t}(q_t)$ for $\ell=1,\ldots,L$.

$$H_{j,n,t} = C_{j,n,t}(q_t,RC_{1,t}) - \sum_{\ell=1}^{2} \sum_{c=0}^{t-1} \sum_{z=n+1}^{t-1} X_{j,\ell,p,c,t}(s_t,W_t,RC_{1,t},q_t)$$

for $j=1,2$, $n=0,\ldots,t-1$.

These equations--together with the quantities $RV_{\ell,t}(q_t)$, $\ell=1,\ldots,L$; $RC_{2,t}(q_t)$; and q_t --jointly define the function $s_{t+1} = g(s_t,W_t,RC_{1,t},q_t)$.

The objective of the firm, then, is to optimally choose W_t and $RC_{1,t}$ to solve

$$f(s_t) = \max \{ -(BC)W_t + \sum_{q_t \text{ in } Q} P(q_t)\{ \sum_{j=1}^{2} CC_j C_{j,t}(q_t,RC_{1,t})$$

$$+ [A_t(q_t) - OC_t(q_t)] \sum_{\ell=1}^{L} V_{\ell,t}(q_t)$$

$$+ \sum_{j=1}^{2} \sum_{\ell=1}^{L} \sum_{p=t-8}^{t-1} \sum_{c=0}^{t-1} [DP_{j,\ell,p,c,t}(RC_{j,p}(q_p),RV_{\ell,c}(q_c)) + DC_j]$$

$$\cdot X_{j,\ell,p,c,t}(s_t,W_t,RC_{1,t},q_t) + af[g(s_t,W_t,RC_{1,t},q_t)]\} \} ,$$

subject to both a capital constraint

$$\sum_{\ell=1}^{L} \sum_{n=1}^{t} U_{\ell,n,t} \leq BL$$

and a coverage constraint

$$\sum_{\ell=1}^{L} \left[\sum_{n=0}^{t-1} U_{\ell,n,t} - \frac{\sum_{n=0}^{t-1} U_{\ell,n,t}}{\sum_{\ell=1}^{L} \sum_{n=0}^{t-1} U_{\ell,n,t}} \sum_{j=1}^{2} \sum_{n=0}^{t-1} H_{j,n,t} I(RV_{\ell,t}(q_t) + \tau - RC_{j,t}(q_t)) \right] \leq CL$$

where τ is an exogenously determined interest rate tolerance, and where

$I(x) = 1$, if $x \geq 0$, and $I(x) = 0$ otherwise.

An Example of the Reduced Model

Given some starting conditions and the policy rules to be followed, the MBF can invoke a computer search routine to solve the auction model. This section presents an illustrative example for the two- and three-period cases using the policy rules described above.

The example considers eight cases, based on what are intuitively the three most sensitive variables. The maximum allowable uncovered position is alternatively $4,000,000 or $20,000,000. Good commitments are determined by the current origination rate plus either .0005 or .0010 (the tolerance factor). If the firm originates too many loans and violates its capital constraint even after exercising all its commitments, additional loans must be sold. To discourage excessive originations, the model assumes such loans are sold at a loss of either .01 or .05. The eight cases represent all possible combinations of these three quantities.

In this example, the MBF has a choice of five yields and six amounts to bid. These yields are presented in Table 1 along with the probabilities of their acceptance. The yields and acceptance probabilities

were derived using data from FNMA auctions during 1974-1978. First a forecast of the low accepted bid is made following Miles and Sears.[15] Then the probabilities of acceptance for bids around this forecast are generated using historical frequencies.

The simulation operated as follows. Before the MBF makes its bid decision in period 1, the low accepted bid in the FNMA acution in period 0 is randomly generated form the distribution described above. This period 0 yield determines the MBF expectations about all the other yields over the relevant horizon in accordance with historical relationships and the experience of the FNMA, the GNMA, and local MBFs.[16] The randomly generated yield also determines the rates the firm may bid in each period (the decision set). Thus, each randomly generated, period 0, low accepted bid provides the input data and decision set used in the eight cases for both the two- and three-period horizons. (Limited computer funds precluded consideration of more numerous possible actions and more than one set of input data.) The dynamic programming technique of backwards optimization determines the appropriate decision for the mortgage banker to make in period 1. [17]

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[15] For a fuller discussion of the econometric model supporting these probabilities, see Miles, Mike E. and Sears, R. Stephen, "An econometric approach to the Federal National Mortgage Association Free Market System Auction," Working Paper, The University of North Carolina at Chapel Hill, School of Business Administration, 1979.

[16] For a discussion of these historical relationships, see Sears, op. cit.

[17] A good introduction to dynamic programming is found in Wagner, Harvey M., Principles of Operations Research, Englewood Cliffs: Prentice-Hall, 1975.

TABLE 1: BID YIELDS AND ASSOCIATED PROBABILITIES OF ACCEPTANCE

PERIOD	YIELD	ACCEPTANCE PROBABILITY
1	.10780	.95
1	.10751	.85
1	.10733	.75
1	.10715	.65
1	.10698	.55
2	.10728	.95
2	.10699	.85
2	.10681	.75
2	.10663	.65
2	.10646	.55
3	.10735	.95
3	.10706	.85
3	.10688	.75
3	.10670	.65
3	.10653	.55

The initial conditions and model parameters are listed in Table 2. The firm begins with a capital constraint of $84,000,000 and a loan portfolio of $80,000,000, of which $72,000,000 is covered by commitments. Its discount rate is set at 10%.

An analysis of the inputs, in conjunction with the random generation of observations from the historical distributions, reveals that interest rates first go down in the second period and then rise slightly in the third period. An examination of the bid yield inputs and the good commitment parameters shows that as the model progresses from period to period, or from auction to negotiated commitment purchase, not all of the commitments purchased in prior auctions continue to qualify as good coverage in certain cases. This captures the principal trade-off of the auction procedure--a higher bid carries a higher probability of acceptance, but may require purchase

E. BID AMOUNTS AND COSTS

Amount	Bid Cost
$ 0	0
$ 3,000,000	.0001
$ 6,000,000	.0003
$ 9,000,000	.0005
$12,000,000	.0007
$15,000,000	.0009

F. NEGOTIATED COMMITMENT RATES

Period	Rate
1	.10707
2	.10655
3	.10662

G. COMMITMENT AND DELIVERY COST PARAMETERS

Type	Commitment Cost	Delivery Cost
FNMA	.0050	.001
Other	.0075	.001

H. LOAN ORIGINATION COSTS AND RATES

Period	Cost	Rate
1	.01	.10720
2	.01	.10668
3	.01	.10675

I. MISCELLANEOUS

Discount rate: .10   (Discount factor = 1/1.10)

Credit line: $44,000,000

Maximum allowable uncovered loans: $4,000,000 or $20,000,000

Allowable premium above origination rate for good commitments:
.0005 or .0010

Forced selling loss percentage: .01 or .05

A. LOAN PORTFOLIO

Amount	Rate
$18,000,000	.10675
$11,000,000	.10685
$22,000,000	.10700
$15,000,000	.10725
$14,000,000	.10745

B. COMMITMENT POSITION

Amount	Rate
$27,500,000	.10675
$28,000,000	.10685
$ 3,000,000	.10730
$ 4,000,000	.10745
$ 9,500,000	.10765

C. EXPIRING COMMITMENTS

Period	Amount	Rate
1	$6,000,000	.10685
1	$4,000,000	.10745
2	$7,500,000	.10675
2	$3,000,000	.10730
3	$9,000,000	.10675
3	$2,000,000	.10765

D. ESTIMATED DEMAND SCHEDULE

Period	Amount	Rate
1	$10,000,000	.10720
2	$11,000,000	.10668
1	$10,500,000	.10675

The \$4 million and \$20 million limits on uncovered loans respectively force a tightening and permit a loosening of the firm's commitment position.

The demand and commitment expiration schedules force the firm in periods 2 and 3 to sell off loans after exercising all commitments. Thus, the impact of the selling loss percentage can be evaluated.

The results of the two-period model are presented in Table 3, while those from the three-period version are found in Table 4. Tables 3 and 4 present the parameter values for each case as well as the values of the decision variables and objective function. The results improve as the restrictions on the firm are relaxed--greater permissable uncovered loan portfolio, more generous definition of good coverage, and lower penalty for excessive loan closings. Other tables, available from the authors, display the conditional portfolio positions of the firm given that the firm ends up in a specific state at the end of the model. These tables give the yields and amounts for the ending loan and commitment portfolios, together with the final coverage exposure.

From the results it appears, at least for the two-period model, that relaxing the coverage constraint produces a wider   in bid yields than does tightening the constraint. It would be hazardous, however, to state this as a firm conclusion without first testing other constraint levels and input data sets.

For at least two cases--exposure limit of \$20 million, good commitment tolerance of .0010 above the origination rate, and both selling discounts --the yields and amounts in the first two periods of the optimal solution to the three-period model differ from the optimal solution to the two-period model. How long an horizon is required before the

fore, remains an open question and one whose solution is a function of a larger computer budget.

TABLE 3

SOLUTION RESULTS FOR THE TWO-PERIOD MODEL
(A and R mean bid accepted and bid rejected, respectively)

CASE	COVERAGE EXPOSURE	SELLING DISCOUNT	COMMITMENT PARAMETER	BID RATE(1) P(A)	BID AMT.(1)	BID RATE (2|A) P(A)	BID RATE(2|R) P(A)	EXPECTED PROFIT
1	$4,000,000	.05	.0005	.10715 / .65	$12,000,000	.10699 / .85	.10699 / .85	$38,221
2	4,000,000	.05	.001	.10751 / .85	15,000,000	.10728 / .95	.10728 / .95	87,828
3	4,000,000	.01	.001	.10751 / .85	15,000,000	.10728 / .95	.10728 / .95	109,281
4	4,000,000	.01	.0005	.10715 / .65	12,000,000	.10699 / .85	.10699 / .85	59,521
5	20,000,000	.05	.0005	.10715 / .65	6,000,000	.10699 / .85	.10699 / .85	154,549
6	20,000,000	.05	.001	.10751 / .85	3,000,000	.10646 / .55	.10728 / .95	190,239
7	20,000,000	.01	.001	.10751 / .85	3,000,000	.10646 / .55	.10728 / .95	211,833

TABLE 4

SOLUTION RESULTS FOR THE THREE-PERIOD MODEL *

CASE	BID RATE (1)	P(A)	AMT	BID RATE (2|A)	P(A)	BID RATE (2|R)	P(A)	BID RATE (3|AA)	P(A)	BID RATE (3|AR)	P(A)	BID RATE (3|RA)	P(A)	BID RATE (3|RR)	P(A)	EXPECTED PROFIT
1	.10715	.65	$12MM	.10699	.85	.10699	.85	.10653	.55	.10653	.55	.10653	.55	.10653	.55	$ 86,975
2	.10751	.85	$15MM	.10728	.95	.10728	.95	.10653	.55	.10735	.95	.10653	.55	.10735	.95	$161,750
3	.10751	.85	$15MM	.10728	.95	.10728	.95	.10653	.55	.10735	.95	.10653	.55	.10735	.95	$202,839
4	.10715	.65	$12MM	.10699	.85	.10699	.85	.10653	.55	.10653	.55	.10653	.55	.10653	.55	$128,049
5	.10715	.65	$ 6MM	.10699	.85	.10699	.85	.10653	.55	.10653	.55	.10653	.55	.10653	.55	$201,498
6	.10751	.85	$ 3MM	.10728	.95	.10728	.95	.10653	.55	.10735	.95	.10653	.55	.10735	.95	$246,426
7	.10751	.85	$ 3MM	.10728	.95	.10728	.95	.10653	.55	.10735	.95	.10653	.55	.10735	.95	$287,585
8	.10715	.65	$ 6MM	.10699	.85	.10699	.85	.10653	.55	.10653	.55	.10653	.55	.10653	.55	$242,572

*AA = accept-accept
 AR = accept-reject
 RA = reject-accept
 RR = reject-reject

SUMMARY AND CONCLUSIONS

This paper has formulated a mathematical model of the mortgage loan place-
ment problem. Although not solvable in itself, given the current state of mathe-
matics, the model provides a conceptual framework for further work in the
field. In particular, smaller, more tractable models of sub-problems can
be extracted from the larger formulation. The paper provides an example
of such a model, one that analyzes the FNMA free market system auction bid-
ding decision in a reduced form dynamic framework. The computer solution
of this auction model is presented for two- and three-period horizons for
a variety of operating parameters and typical input data.